Are Average Growth Rate and Volatility Related?

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Abstract

The empirical relationship between the average growth rate and volatility of growth rates has important policy implications, which depend critically on the sign of the relationship. Following Ramey and Ramey (1995), who study the relationship across countries for the period 1962–1985, a wide consensus has been building, that the correlation is negative. We replicate their result, and then test the relationship in a more recent version of the dataset with more countries and longer periods. Further, we exhaustively investigate this relationship in alternative datasets, for various definitions of growth rates, across time, across countries, and within groups of countries and find that negative correlation between average growth and volatility is not robust to either the composition of the sample, or the definition of growth rates. Our analysis suggests that there is no significant relationship between the two variables in question.

Keywords: Volatility; Growth; Fluctuations
JEL Codes: E32, O40.

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1 Introduction

The policy implications of the relationship between average growth rate and volatility of growth rates are significant, and, moreover, depend on the sign of the relationship. A negative relationship between the average growth rate and volatility of growth rates would imply that policies that reduce short run movements in output will also increase the long term growth rate. In fact the belief that the two are negatively related is one of the main justifications for short run "stabilization" policies, which often refer to policies aimed at reducing volatility.[1] The World Bank and the IMF routinely advise governments to reduce fluctuations to achieve higher growth rates.[2] The calculation of the welfare cost of volatility will also be higher if this negative relationship is taken into consideration.

In the empirical literature researchers have found both positive and negative relationships between the two variables. Among the recent studies, Ramey and Ramey (1995) find a negative relationship in their analysis for the period 1962–1985. Now data is available for a longer period of time and for more countries. The objective of this paper is to investigate if there is any consistent and significant

[1]Note that here we are not trying to ask whether reducing volatility is worthwhile. Volatility may have other effects, particularly welfare effects, which might justify policies aimed at managing volatility. What we are pointing out here is that one of the main justifications of such policies is that reducing volatility increases average growth and our intention is to take a close look at that justification. Also note that in this paper we are not interested in the direction of causality, but in the literature it is more commonly considered that volatility effects growth rate.

[2]A large number of working papers and economic reports published by the World Bank and IMF recommend reducing volatility to achieve a higher growth rate. For example, in an Economic Report "Brazil - Stability for Growth and Poverty Reduction" (World Bank, 2003), published by the World Bank, it says that "... even short run volatility, ..., can have persistent effects on growth."

relationship between average growth rate and volatility of growth rate for a wide range of datasets and for several definitions of growth rates.

Ramey and Ramey (1995) finds that the average growth rate decreases as volatility of growth rates increases. They draw their conclusion using data from 92 countries for the period 1962–1985 and also separately from a data set of OECD countries for the period 1952–1988. In contrast, in an earlier study using a set of 47 countries for the period 1950–1977, Kormendi and Meguire (1985) found that average annual growth rates were positively related to the volatility of growth rates. Grier and Tullock (1989) corroborate the Kormendi and Meguire (1985) result using a sample of 113 countries for the period 1950–1981.

Other, more recent papers, that have looked into this issue include Aghion et. al. (2005), who study how uncertainty and credit constraints effect growth and volatility, and Kose et. al. (2006), who investigate how the relationship has changed over the decades and how financial integration effects volatility.

In this paper, we address the robustness of the relationship between the average growth rate and volatility of growth rates. There are two dimensions along which we test their results. First, we test robustness of the result in different datasets — we use a larger dataset, multiple sources of data, a longer time-period, different subsets of the data and different time-periods. We also use time series data to study the relationship.

On another dimension, we examine if the definition of growth rate matters. Ramey and Ramey (1995), have used the log difference of GDP per capita in consecutive years as the definition of growth rates. The average growth rate is calculated as the mean of those year-to-year growth rates over the relevant period and the volatility as the standard deviation of the same. However, the year-to-year growth rate, as well as the average, can be calculated in alternative ways. It would be assuring if the relationship was consistent across the definitions and did not depend on the choice of the definition. Hence, we check if the relationship is stable across various definitions. For this purpose we redo the regressions in Ramey and Ramey (1995), using their sample, with other definitions of growth rates (year-to-year and average). Further, we test the relationship using each of the various definitions of growth rate in all the samples that we use . We also show analytically that the log definition will tend to bias the estimated correlation between average growth rate and standard deviation compared to some other definition of growth rate.

Methodologically we follow Ramey and Ramey (1995) for most of the paper. For all samples, we first run OLS regressions between average growth rate and volatility without any control variables. We then add control variables and estimate the coefficient in two different ways. First, we use the growth rates that vary across years and country to form a panel and use MLE for the estimation, exactly like in Ramey and Ramey (1995). This helps us to replicate their results and compare other results with those in their paper and ensure that the differences in the results are not due to any methodological difference. We also estimate the coefficient using a simple OLS on the cross-country data with control variables.

Our analysis brings out fresh doubts about the relationship—we fail to find a robust significant relationship between average growth rate and volatility of growth rates.

The paper is organized as follows: The next section presents a simple non-parametric test of the relationship followed by the section where we check for robustness across various datasets. In section 4, we use time series data. In section

5 we discuss the role played by the definition of growth rate. Finally we conclude.

2 A simple exercise

To begin with we do a simple and intuitive exercise. Assume that the average growth rate and the volatility of growth rates are related, positively or negatively. Now, if we have two groups of countries such that, on an average, the mean growth rates are different across groups, then the average volatilities of those two groups must also be different.

We use data from the Penn World Tables (PWT) 6.1. and divide the sample of 109 countries in two groups based on the average growth rate for the period between 1960 and 1996. We order the countries according to their average growth rates [3] in that period and put the top 40% of the countries in the first group. We call these "high growth countries". The second group consists of the bottom 40% of the countries, referred to as "low growth countries". The middle 20% of the countries are discarded so that there is a clear difference between the two groups. The average growth rate for the low growth rate countries is 0.0027 while the average growth rate for the high growth countries is 0.0378, which is higher by a factor of 14. Now, if average growth rate and volatility are related then we would expect the volatilities to be significantly different for these two groups of countries given that the growth rates are different.

[3] In this exercise, for each country, we calculate the annual growth rate of GDP per capita for each year, $g_t = \frac{y_t - y_{t-1}}{y_{t-1}}$, and then take the arithmetic mean as the average growth rate. Volatility is the standard deviation of those annual growth rates.

Table 1: Volatility Across Groups with Different Growth Rates

	Mean of Average Growth Rates		*Mean Volatility*	
	Low Growth Countries	High Growth Countries	Low Growth Countries	High Growth Countries
All	0.0027	0.0378	0.0595	0.0527
Poor	-0.0013	0.0397	0.0663	0.0635
Rich	0.0091	0.0372	0.0441	0.0466

However, from the first row of Table 1 we find that there is a meager difference between the mean volatilities of these two groups of countries — the mean standard deviation for low growth countries is just 1.1 times that for the high growth countries.

We repeat the exercise, but now control for the wide income differences across countries. We first divide all countries according to their initial income (real GDP per capita in 1961). The poorest 40% of the countries were included in the "poor group" (initial income less than $1694.00), while the richest 40% of the countries made up the "rich group" (initial income greater than $2776.7). Each group consists of 44 countries. Then within each group we divide the countries according to their growth rates as described earlier.

From the last two rows of table 1 we can see that the results for both the groups, poor and rich, are similar to what we have found earlier. In both groups the average growth rates across low growth countries and high growth countries differ substantially but the mean volatilities across them are quite similar.

This simple exercise plants a seed of doubt about whether there is a systematic relationship between the average growth rate and volatility of growth rates.

3 Robustness of the relationship across datasets

In this section we explore whether the relationship between average growth rate and volatility is robust to the choice of dataset. To that end we run two sets of regressions on all the datasets for each definition of growth rate, one without any control and one with controls.

3.1 Datasets

Here we list the various datasets we use to check for the relationship between average growth rate and volatility.

3.1.1 PWT 5.0

The first dataset that we use is the exact dataset used by Ramey and Ramey (1995). The data is originally from PWT 5.0, but the data used here is downloaded from Valerie Ramey's website.[4] The sample has data on 92 countries for 1962-1985.

3.1.2 PWT 6.1

The next sample that we use consists of all countries that we could get data on from the latest version of Penn World Tables, PWT 6.1. The PWT 6.1 provides data on a larger set of countries and for a longer time period than PWT 5.0. We not only regress average growth rate on volatility for the longest period for which data is available (1962-2000), [5] but also on two subsets, 1962-1985 and 1986-2000.

[4] http://www.econ.ucsd.edu/ vramey/research/volat/volat.html.

[5] 1962-2000 is the range for the growth rates, so the data actually ranges from 1961-2000. In all other cases too, the sample period in the text refers to the years for which growth rate data has been used.

We also run all of the regressions on a set of countries that exclude oil exporters [6], but do not report those since the results are similar.

3.1.3 IFS

The PWT 6.1 provides data for a large set of countries for a long period of time and hence it is extremely useful for our analysis. The PWT provides data in a common currency, which is a necessary requirement for many research agendas. Since we are only interested in growth rates, data on GDP per capita in local currency would be sufficient. In fact, it would avoid any problems in the data that may creep in while converting from local currency to US dollars. International Financial Statistics (IFS) published by the IMF provides data on GDP per capita in local currency. So we also use that data for our regressions. The problem is, however, the data is not as comprehensive as the PWT 6.1. The largest set of countries we could get data on is 75, for the period 1986-2000. We report results from regressions for three different periods — 1962-1985, 1986-2000 and 1971-2000 (which is the longest period for which continuous data is available for a reasonable number of countries).

Apart from cross-country data we also check for the relationship in the data on US states. For brevity, however, we omit those results, which also support our conclusions, from the paper.

3.2 Definitions of growth rate

There are various ways in which the growth rate, year-to-year (annual) as well as the average, can be calculated. The volatility is measured as the standard devia-

[6] Dummy for oil exporting countries taken from Easterly and Kraay (2000).

tion of the annual growth rates, so this will also depend on the definition used to calculate the annual growth rates. To ensure that our results are not dictated by the choice of a particular definition of growth rate, for each of the samples we run the regressions using various definitions of growth rates. For the samples consisting of all countries across the world, we report the regression results for each of the following cases:

Case 1: Log definition: $g_t^L = log(y_t/y_{t-1})$. The average growth rate and the volatility of growth rates are calculated as the mean and standard deviation, respectively, of annual growth rates calculated using the above formula.

Case 2: Standard[7] definition: $g_t = (y_t - y_{t-1})/y_{t-1}$. Again, the average growth rate and the volatility of growth rates are calculated as the mean and standard deviation, respectively, of annual growth rates calculated using the above formula.

Case 3: Geometric definition: $= (\frac{Y_T}{Y_0})^{1/T} - 1$. This does not give us annual growth rates, but the average growth rate over a period of time. When using this definition for average growth rate, the volatility is taken as the standard deviation of the annual growth rates calculated using the standard definition.

Case 4: OLS definition: The average growth rate obtained as the coefficient in an OLS regression of log GDP per capita on time. The volatility in this case, also, is taken as the standard deviation of the annual growth rates calculated using the standard definition.

3.3 Methodology

For most part, we follow the estimation strategy used in Ramey and Ramey (1995). This helps us to replicate their results for the sample used in their paper, for both, regressions with and without controls, and ensures that the differences that we obtain with other samples and definitions of growth rates is not due to change in methodology.

The regression equation without any controls is given by:

$$\overline{g}_i = \alpha + \beta\sigma_i + \varepsilon_i, \tag{1}$$

where $\overline{g}_i$ represents the average growth rate (for any definition of growth rate used) in country i for the given period. The measure for volatility in a country i is the standard deviation of annual growth rates (measured by either log or standard definitions, depending on the case) in that period , σ_i.

For the second set of regressions we use various controls as independent variables in the regressions. Ramey and Ramey (1995) use the following set of modified Levine-Renelt (1992) control variables:

- average investment fraction of GDP
- average population growth rate
- initial human capital
- initial per capita GDP (in log terms)

Kormendi and Meguire (1985) have also used a similar set of instruments. Following these papers we use the same set of controls in all datasets considered here. Data on all variables, except human capital, are from PWT 6.1. As a proxy for initial human capital, in most of the samples we use the average schooling years in the total population over age 25 in 1960. However, for the sample which consists only of the OECD countries we use total gross enrollment ratio for secondary education in 1960 (also following Ramey and Ramey (1995)). Data for both of these variables are from Barro-Lee (1996, 2001) data set. [8]

For the analysis with control variables we use two different methods to estimate the relationship between average growth rate and volatility.

The first estimation strategy is the one used in Ramey and Ramey (1995). This involves taking observations on growth rates for each year for each country and regressing it on a measure of country level volatility (standard deviation of growth rates) and country level controls. The advantage of this procedure is that it allows us to exploit the panel variation in a larger sample than the cross-sectional framework allows. Further, using this method helps us to compare our results with those in Ramey and Ramey (1995).

This method of estimation is described by the following equations.

$$g_{y_{it}} = \alpha\sigma_{y_i} + \beta\mathbf{X_i} + \epsilon_{it} \tag{2}$$

$$\epsilon_{it} \sim N(0, \sigma_i^2), \quad i = 1, \cdots, I; \quad t = 1, \cdots, T \tag{3}$$

where $g_{y_{it}}$ is the growth rate of country i at time t and σ_{y_i} is the standard deviation of the growth rate for the time period 1 to T. X_i is the vector of control variables (including a constant). We use MLE to estimate the coefficients.

In this case, however, we can only use two definitions of growth rate — the log difference and the standard definition, since here we exploit the year-to-year

[8]Downloaded from http://www.nuff.ox.ac.uk/Economics/Growth/barlee.htm

variations. The geometric definition and the OLS definition gives us only the average growth rate over the years.

To accommodate all four definitions of average growth rate we use a second method. We run a simple linear regression on cross-country data. Thus we estimate the following equation using OLS:

$$\overline{g}_i = \alpha + \beta\sigma_i + \gamma\mathbf{X_i} + \varepsilon_i, \tag{4}$$

where $\mathbf{X_i}$ is the vector of control variables for country i.

In all regressions we exclude those countries as outliers for which the volatility of growth rates is more than four standard deviation away from the average volatility for all countries in the sample as outliers.[9]

3.4 Regression results

The results of the various regressions for the sample of all countries with full set of data are presented in Table 2. The table gives the results of the regressions without controls as well as regressions with controls, for both the panel (MLE) and simple cross-country (OLS) regressions. In the regressions with controls, since our focus is solely on the significance of volatility, we do not report the coefficients for the rest of the independent variables included in the regressions. (These are available in the older working paper version and can be requested from the authors.)

We observe from the table that a majority of the coefficients, both with and without controls, are insignificant. For regressions without controls, for the sample used in Ramey and Ramey (1995) (PWT 5.0, 1962-1985), the coefficient is

[9]If we include the outliers in the regressions, the coefficient on volatility is insignificant more often.

significant at 5% confidence level only for the log definition, significant at 10%, but not 5%, for the geometric definition and insignificant for the other two definitions. Further, for the same years, when we use data from PWT 6.1 for a larger number of countries, the coefficient is never significant. In the regressions using IFS data the coefficient is not significant even once. For regressions in which control variables are included, we again get a large number of cases where the coefficient is insignificant. The coefficient is more often insignificant when we run OLS regression on the cross-country data than when we estimate the coefficient using MLE on a panel data. Also, considering all regressions, the coefficient is significant more times for the log definition than any other definition (there are many cases for which the coefficient is insignificant for the log definition as well). Clearly, whether or not the coefficient is significant depends on the time-period as well as the definition of average growth rate chosen. In most cases, though, it is insignificant.

Table 2: Average Growth Vs. Volatility Regressions: World

			Standard Definition		Log Definition		Geometric Definition		OLS Definition	
Source	Period	Countries	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
Regressions without Control Variables (OLS)										
PWT 5.0	1962-1985	92	-0.0604	-0.8846	**-0.1535**	-2.3366	-0.1318∗	-1.9355	-0.160	-1.6033
	1962-1985	112	0.0423	0.6862	-0.0585	-0.9447	-0.0281	-0.4644	-0.0529	-0.8513
PWT 6.1	1986-2000	107	-0.1392	-1.9952	**-0.2200**	-3.4089	**-0.1755**	-2.6917	**-0.2058**	-2.7196
	1962-2000	98	-0.0725	-1.3227	**-0.1561**	-2.9098	**-0.1269**	-2.3910	**-0.1296**	-2.1770
	1971-2000	51	0.0538	0.7145	-0.0692	-0.7427	-0.0587	-0.7168	-0.1134	-1.4061
IFS	1962-1985	34	-0.1159	-0.6239	-0.2099	-1.1326	-0.1514	-0.8376	-0.2091	-1.0468
	1986-2000	75	0.0800	0.9400	-0.0639	-0.7677	0.1221	0.9469	-0.0174	-0.1959
Panel Regressions with Control Variables (MLE)										
PWT 5.0	1962-1985	92	**-0.2110**	-3.0644	-0.0800	-1.1614				
	1962-1985	83	**-0.1224**	-2.1222	**-0.1850**	-3.2236				
PWT 6.1	1986-2000	78	-0.0508	-0.7389	-0.0984	-1.4486				
	1962-2000	75	-0.0873	-1.6379	**-0.1464**	-2.7580				
	1971-2000	34	-0.1734∗	-1.8941	**-0.3190**	-3.2448				
IFS	1962-1985	27	0.1613	1.1008	0.0351	0.2454				
	1986-2000	50	**-0.3412**	-3.7867	**-0.3703**	-4.1536				
Cross-Country Regressions with Control Variables (OLS)										
PWT 5.0	1962-1985	92	-0.0062	-0.0837	-0.1113	-1.5655	-0.0822	-1.1149	-0.0781	-0.9931
	1962-1985	83	0.0446	0.5959	-0.0428	-0.5801	0.0095	0.1263	-0.0237	-0.2904
PWT 6.1	1986-2000	79	-0.0833	-0.9611	**-0.1685**	-2.0630	-0.1314	-1.5655	-0.1610	-1.6108
	1962-2000	75	-0.0175	-0.2332	-0.1031	-1.4077	-0.0714	-0.9696	-0.1360	-1.5559
	1971-2000	34	0.1338∗	1.8409	0.0089	0.0886	-0.0627	-0.5510	-0.0464	-0.5537
IFS	1962-1985	27	0.1422	0.8756	0.0549	0.3519	0.0677	0.4302	0.0794	0.4265
	1986-2000	50	-0.1848	-1.5073	**-0.2527**	-2.1935	0.0392	0.1210	**-0.2767**	-2.0122

Coefficients that are significant at 5% confidence level are marked in bold. ∗ signifies insignificant at 5%, but significant at 10% confidence level.

The coefficients on the control variables are not reported here. They are available in an older Working Paper version of the paper or by request from the authors.

3.5 Relationship within groups of countries

In all the regressions in the previous sections we had chosen a sample of all countries for which data was available. Now we split the available data to subsets of countries which share similarities in some dimension and look for the robustness of the relationship within each group[10]. For brevity we report regression results for the log definition and standard definition only. For other cases, the results were similar.

The first group of countries in our analysis are the OECD countries.

3.5.1 OECD

The sample includes the 24 countries (23 countries in some sub-samples due to the reunification of Germany) that were part of the OECD before 1990. Table 3 provides the results from the various regressions.

From the table we find that in this sample the coefficient on volatility is always positive, though never significant at 5% level. The results are similar if we include all the present OECD members.

[10]We also tried using dummy variables instead of splitting the sample. The results were similar, but the distributions of the error terms across the subsamples are not homogenous, so we decided to persist with the split-sample analysis.

Table 3: Average Growth Vs. Volatility Regressions: OECD Countries

	Standard definition		Log definition	
Period	Coeff.	t-stat	Coeff.	t-stat
	Regressions without Control Variables (OLS)			
1962-1985	0.3226	1.5575	0.2465	1.1810
1986-2000	0.4637*	1.7168	0.3728	1.3886
1962-2000	0.3572*	1.8310	0.2902	1.4593
	Panel Regressions with Control Variables (MLE)			
1962-1985	0.1298	0.7455	0.0885	0.5103
1986-2000	0.2252	1.1713	0.1523	0.8110
1962-2000	0.1407	0.9267	0.0892	0.5873

Data: PWT 6.1 and Barro-Lee data set.

Coefficients that are significant at 5% confidence level are marked in bold.

$*$ signifies insignificant at 5%, but significant at 10% confidence level.

3.5.2 Geographically separated groups

Next we divide all countries by their geographic region and look for patterns within each region. We regress average growth rate on volatility of growth rates for each of the groups. Table 4 reports results from the regressions without control variables only for cases in which the regression coefficient is significant. For all other cases (regions or time-periods) the coefficient is insignificant.[11]

From the table we observe that we did not find a negative relationship even in

[11] In PWT 6.1 data, North Africa is grouped along with Middle East, so while analyzing just African Countries (and the complimentary set) we did the analysis twice, first we took all African countries except the North African countries and second, we took all African countries plus the Middle Eastern countries. The results are quite similar.

Table 4: Regions where the coefficient is significant

Sign	Region	Period	Definition of gr. rate
	Africa	1962-1985	Standard only
Positive	West Europe	1962-2000	Standard,log at 10%
	West Europe, Canada & US	1962-2000	Standard,log at 10%
Negative	None	All Periods	Standard,log

Data: PWT 6.1

one group. Also it is interesting to notice which groups have a significant positive relationship—Africa and West Europe, with or without North America, (virtually the OECD countries)—two very different group of countries, not only in terms of income levels, but also in terms of political structures.

With the control variables included in the regressions, the coefficient on volatility is insignificant for all regions.

3.5.3 Groups according to political structure

We also divide countries according to the political structure of the country and then test for the relationship within each group of similar countries.

The data on political structure is from the Polity III data by Jaggers and Gurr (1996). [12] We divide the countries in to two groups, "Democracies" and "Non-Democracies" using the Polity III data. The Polity III data provides a score for *democracy* for each country for each period. We add up democracy scores for each country over all years (1960-1994) and classify a country as a non-democracy if the sum is below certain cut-off.[13] We have 61 countries classified

[12] We also use another data, the Gastil Scale published by Freedom House, instead of the Polity III data and do the whole analysis again. The results are very similar.

[13] The maximum possible score for any year is 10, so for 35 years a sum of 350 is the maximum possible. We set the cut-off at 150.

as non-democracies (42 if data till 2000 is used) and 45 democratic countries (42 if data till 2000 is used).

Then we run regressions between the two variables of interest for each group, for each sample period. None of the regression coefficients in these regressions are significant for the standard definition and only one is significant for the log definition. In some cases the coefficients are positive, though insignificant.

Adding the various control variables in the regressions we find non-democracies have significant negative coefficient for a few sample periods, while the rest are insignificant.

Table 5: Growth vs. Volatility Regression: Democracies

			Standard definition		Log definition	
	Period	Countries	Coeff.	t-stat	Coeff.	t-stat
Democracies	1962-1985	45	0.1669	1.4303	0.0949	0.7976
	1986-2000	42	-0.1399	-0.8816	-0.2069	-1.3403
	1962-2000	42	-0.1117	-0.8756	-0.1767	-1.3778
Non-Democracies	1962-1985	61	0.1260	1.4995	0.0202	0.2366
	1986-2000	54	-0.1022	-1.1828	**-0.1844**	**-2.3401**
	1962-2000	52	0.0204	0.2882	-0.0715	-1.0219

Data: PWT 6.1 and Polity III

4 Relationship in time-series data

So far we have been using cross-section data. We now probe the relationship using time series data provided by Angus Maddison at his website [14].

We divide the available data in non-intersecting five year periods (like 1920-1924, 1925-1929).[15] For each country we run a regression of average growth rate against volatility calculated for each five year period.

The results are summarized in the table 6. The coefficient on the volatility is insignificant for a vast majority of the countries, negatively significant for a few and positively significant for yet another few countries. Thus, there is no conclusive evidence of any relationship between the two variables of interest, within countries over time either.

Table 6: Time Series Results

		Number of Countries					
Period	Total	Negative Significant		Positive Significant		Insignificant	
		Standard	Log	Standard	Log	Standard	Log
1870-2001	22	6	8	0	0	16	14
1900-2001	29	9	13	1	0	19	16
1950-2001	137	20	22	7	5	110	110

Data: http://www.eco.rug.nl/ Maddison

[14]http://www.eco.rug.nl/ Maddison

[15]We have also tried five year rolling windows. Results are similar.

5 Does definition matter?

In the previous sections we observed that often the significance, and sometimes the sign, of the relationship between average growth rate and volatility hinged on the choice of the definition of growth rate. In this section we discuss why the choice of the definition can influence the result. [16]

Note that only two definitions, standard growth rate and log difference, give us growth rates for each year, which can then be used to calculate the average as well as the standard deviation. So, we start our discussion by comparing these two.

Notice, the two definitions are related. We can expand log to get,

$$g_t^L = \log(1 + g_t) = g_t - \frac{1}{2}g_t^2 + \frac{1}{3}g_t^3 - \cdots = g_t - e_t, \tag{5}$$

where, $e_t = \frac{1}{2}g_t^2 - \frac{1}{3}g_t^3 + \cdots$. The error term, e_t, is small when growth rates are near zero and the two definitions are close. However, as g_t increases, e_t is not insignificant. The log function being a strictly concave function, "squeezes" higher growth rates more than low growth rates. Thus, the volatility of growth rates of countries which tend to have high growth rates across time will be lower when the log approximation is used to measure the growth rate than when the standard definition is used.

A more rigorous demonstration that the log definition, compared to the standard definition, is more likely to lead to the finding of a negative relationship between average growth rate and volatility of growth rate follows.

[16] We would like to make it very clear that it is not our intention in this paper to comment on whether any definition is superior to the others. In many cases the choice will depend on the model at hand. Here we do not in anyway order the various definitions in any form of desirability. We are purely interested in the statistical relationship between the two variables, irrespective of the definition chosen.

Suppose there are two countries 1 and 2 which have different expected growth rates, (defined as $g_t = (y_t - y_{t-1})/y_{t-1}$) but the same standard deviation of the growth rates. More specifically, suppose that growth rate each period in country 1 and 2 are independent draws from two different distributions which are identical up to the addition of a positive constant. That is, assume that the growth rates in country 1 are distributed as a random variable X with a well defined expected value on $[-1, \infty)$ and, a positive and finite variance and those in country 2 are distributed as the random variable Z such that $Z = X + a$, where $a > 0$ is a constant. By construction $var(X) = var(Z)$ and $E(Z) > E(X)$. That is country 2 has a higher average growth rate than country 1 but the same volatility of growth rates when measured using the standard definition. We want to show that $var\left(\ln\left(1+Z\right)\right) < var(\ln(1+X))$.

Proof.

$$var[\ln\left(1+Z\right)] - var[\ln(1+X)]$$
$$= E[\ln(1+X+a) - E[\ln(1+X+a)]]^2 - E[\ln(1+X) - E[\ln(1+X)]]^2$$

Define $\widehat{x}$ as the value in $[-1, \infty)$ such that $\ln(1+\widehat{x}) = E[\ln(1+X)]$. We can transform the above difference of variances in the following way:

$$\begin{aligned}
& E[\ln(1+X+a)-E[\ln(1+X+a)]]^2 - E[\ln(1+X)-E[\ln(1+X)]]^2 \\
= \; & E[\ln(1+X+a)-\ln(1+\widehat{x}+a)+\ln(1+\widehat{x}+a)-E[\ln(1+X+a)]]^2 \\
& -E[\ln(1+X)-\ln(1+\widehat{x})+\ln(1+\widehat{x})-E[\ln(1+X)]]^2 \\
= \; & E[\ln(1+X+a)-\ln(1+\widehat{x}+a)]^2 + E[\ln(1+\widehat{x}+a)-E[\ln(1+X+a)]]^2 \\
& +2E[(\ln(1+X+a)-\ln(1+\widehat{x}+a))(\ln(1+\widehat{x}+a)-E[\ln(1+X+a)])] \\
& -E[\ln(1+X)-\ln(1+\widehat{x})]^2 \\
= \; & E[\ln(1+X+a)-\ln(1+\widehat{x}+a)]^2 - (\ln(1+\widehat{x}+a)-E[\ln(1+X+a)])^2 \\
& -E[\ln(1+X)-\ln(1+\widehat{x})]^2 \\
= \; & E[\ln(1+X+a)-\ln(1+\widehat{x}+a)]^2 - E[\ln(1+X)-\ln(1+\widehat{x})]^2 \\
& -(\ln(1+\widehat{x}+a)-E[\ln(1+X+a)])^2
\end{aligned}$$

by concavity and monotonicity of the log function, $\forall x \geq -1$ we have $|\ln(1+x+a)-\ln(1+\widehat{x}+a)| \leq |\ln(1+x)-\ln(1+\widehat{x})|$, with strict inequality for any $x \neq \widehat{x}$. Thus we have:

$$E[(\ln(1+X+a)-\ln(1+\widehat{x}+a))^2 - (\ln(1+X)-\ln(1+\widehat{x}))^2] < 0.$$

Hence,

$$var\left(\ln\left(1+Z\right)\right) - var(\ln(1+X)) < 0. \qquad \blacksquare$$

Thus, for two countries for which the distribution of growth rates are identical up to the addition of a positive constant, the country with a higher average growth rate will have lower variance when the log definition is used. This can be easily generalized to N countries.

This shows that use of log approximation as a measure of growth rates will create a bias towards finding a negative relationship between the average growth rate and volatility of growth rates when compared with the analysis where the standard definition is used.

It is also illustrative to compare the average growth rates. Compared to geometric definition the average of annual growth rates using the standard definition is likely to overestimate the average growth rate over that time period, particularly for countries which might have experienced significant negative growth. On the other hand, the use of average of log differences will underestimate the average over the time period, which can be significant for countries experiencing high growth rates.[17]

These differences in measuring average growth rate and volatility, depending on the choice of definition, manifests itself in the regression results. Also, driven by the fact that the choice of definition can change the result, we opted to report the regression coefficient for a variety of definitions. However, there was no consistent relationship observed for any definition, though there were more number of significant negative cases when the log definition was used, in line with the discussion above.

6 Conclusion

The central question this study addresses is whether there is a relationship between the average growth rate and the volatility of the growth rates. We tested

[17] To see why log difference will underestimate, consider this: Geometric mean $= (\frac{y_T}{y_0})^{1/T} - 1 = Z - 1$, where, $Z = (\frac{y_T}{y_0})^{1/T}$. Average of log difference $= \frac{1}{T}\log(\frac{Y_T}{Y_0}) = \log(Z)$. It can be easily shown that the difference is $\log(Z) - (Z-1) \leq 0$.

the relationship along two dimensions: one, whether the relationship is consistent across data sets and time periods, and two, whether the choice of definition of growth rate matters.

We also tested the relationship across data sets and time periods using data from Penn World Tables and International Financial Statistics and within various subgroups of countries. We found that often the relationship was not significant, with or without controls, for any definition of growth rates. The number of cases where we found a negative significant relationship was higher for the log definition. There were a few cases with positive significance. An intriguing observation was that two very different groups of countries that had positive significance, Africa and OECD. The levels of volatility across the two groups are also quite different and it would be interesting to examine the reason for this difference.

Using time series data the relationship was sometimes negatively significant and sometimes positively significant under both the definitions, but an overwhelmingly large number of regressions produced insignificant coefficients.

To test the importance of the definition of growth rates, we regressed average growth rates on volatility for various definitions of growth rates—the standard growth rate, the log difference, the geometric definition and the OLS definition—on each world-wide sample. In doing this, our objective is not to comment on which definition of growth rate is better, but to examine if there was a consistent relationship for any definition of growth rate. What we observed is that sometimes the significance of the relationship depended on the definition. An interesting case was when we used exactly the same sample as Ramey and Ramey (1995) and ran regressions with various definitions of growth rates. There was a significant negative relationship at 5% confidence level, as in Ramey and Ramey (1995), only when we used the log definition. For the geometric definition, it was significant at 10% confidence level and insignificant for the two other definitions. There are two definitions, standard and log difference, that give us growth rates for each year (as opposed to average growth rates), which can then be used to calculate the standard deviation. We showed mathematically how the use of log difference can create a bias towards finding a negative relationship even when a relationship is absent if the standard definition is used.

Thus, we establish that the relationship depends on the choice of dataset, sample period and definition of growth rate. For a large number of samples the relationship between average growth rate and volatility of growth rates is non-existent, irrespective of the definition of growth rate chosen, including log differences. We also find the use of the log definition for growth rates may create a bias towards finding a negative relationship between average growth rates and volatility of growth rates.

To conclude, overall, we do not find a consistent relationship between the average growth rate and volatility of growth rates.

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