Do firms value knowledge spillover? A model of R&D competition, endogenous R&D appropriability and product differentiation*

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Abstract

This paper proposes a model of R&D competition with endogenous spillover. Two competing firms in a standard Hotelling model are allowed to conduct process R&D prior to price competition. Their product locations affect not only the level of product differentiation, but also their ability to realize knowledge spillover. If the firms choose to develop more homogeneous products, they will face more intense price competition, but will also be able to appropriate additional R&D from their rivals. The equilibrium level of differentiation depends on the extent of consumer heterogeneity, the cost of R&D and the sensitivity of the spillover rate to the level of differentiation, among other factors. For sufficiently heterogeneous consumers and moderately high R&D costs, there exists a symmetric equilibrium with moderate product differentiation and moderate information flow. This result runs in contrast to previous literature that predicts either maximal or minimal inter-firm information flow in Cournot setting. The results also yield interesting welfare and policy implications.

Keywords: Spillover, product differentiation, R&D, consumer heterogeneity

JEL classification: D21, L11, L13.

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1 Introduction

Knowledge spillover is a ubiquitous phenomenon in the industrial world. It has long been recognized that firms are often able to appropriate the gains from R&D activities undertaken by their rivals through technological spillover. Spillover can arise from numerous sources. For instance, a firm could derive useful, although limited, insights from scientific publications by scientists in rival firms. Alternatively, it could obtain significant technical know-how by conducting reverse engineering, or simply by hiring experienced staff from its competitors.

Over the past two decades, a wealth of theoretical and empirical literature has emerged that sheds light on competing firms' R&D activities in the presence of knowledge spillover. The issue of firm-level spillover has been discussed notably by Griliches (1979) and later by Cohen and Levinthal (1990), among others. d'Aspremont and Jacquemin (1988, 1990) and Kamien, Muller and Zang (1992) formalize the notion of spillover or R&D appropriability in the context of oligopolistic product market competition. These papers investigate firms' strategic R&D activities. They usually regard spillover as the "manna from heaven" and assume that a fixed portion of every firm's process R&D effort *leaks* and reduces the other firms' cost.

Firms' ability to realize beneficial information flow can depend crucially on many environmental factors that are exogenous to their own autonomous actions, such as the strength of the intellectual property (IP) regime, the characteristics of the market environment, etc. However, it has been increasingly recognized in the literature that the level of information spillover could partly result from firms' strategic choices. A handful of recent studies have argued that the level of R&D appropriability can be determined endogenously by firms' autonomous actions. The current paper embraces this notion and attempts to investigate activities of firms when their strategic choices affect their ability to realize knowledge spillover from rivals.

Cohen and Levinthal (1990), for instance, have argued that a firm's own R&D effort not only enhances its manufacturing productivity, but also improves its capability to utilize and assimilate the knowledge spilled over from others. This is commonly referred to as the "absorptive capacity" of a firm. Kamien and Zang (2000) integrate the notion of absorptive capacity into the existing framework of R&D with spillover. The absorptive capacity of a firm depends on the *connectedness* of its research track with that of other rival firms. Thus, the level of absorptive capacity of a firm is determined by firms' strategic choices. Specifically, the ability of the firms to appropriate R&D spillover is maximized when they choose common research tracks, while it is minimized when firms choose completely idiosyncratic approaches. In the context of competitive R&D, Kamien and Zang (2000) have shown that duopolistic firms tend to minimize research connectedness. Wiethaus (2005) further elaborates on the role of connectedness in determining inter-firm information flow. His paper, in contrast to Kamien and Zang (2000), shows that firms maximize connectedness by adopting identical research approaches in equilibrium.[1]

This paper continues the research effort to investigate firms' strategic choice of research connectedness and the resulting level of R&D appropriability (or effective knowledge spillover). We attempt to analyze the trade-off that competing firms face when they choose similar research approaches. In contrast to the existing literature, here a firm's research track is not treated as a choice that is independent of the firm's other actions. A firm's choice of research approach is related to its strategic plan of product configuration. More specifically, each firm is allowed to design its product before subsequently conducting R&D activity and marketing the product. The impact of this decision is two-fold: it determines the extent of product differentiation and, through this, affects the ability of the firm to obtain beneficial knowledge spillover. To put it simply, distinctly differentiated products restrict knowledge spillover, while more homogeneous products allow firms to take advantage of useful information flow from one another.

It is intuitive that a higher level of spillover causes firms to free-ride on their rivals' R&D. This immediately creates the following tension. More differentiated products soften price competition on the one hand, while spurring R&D competition on the other, as this restricts inter-firm spillover. Does profit maximization dictate that they should be maximally connected in terms of their research approaches? Or does profit maximization lead to

[1] Weithaus (2005) conducted a numerical exercise on Kamien and Zang's model and reported that the exercise yielded two equilibria — one where firms chose different R&D strategies and one where firms chose "broadly similar" R&D strategies

maximum product differentiation and minimal information flow?

Existing empirical and anecdotal evidence tend to support competing views. Liao, et al (2007) have looked at data from Taiwanese firms (from the electronic, financial insurance and medical industries) and found that these firms engage in knowledge sharing, which in turn increases their absorptive capacity. Evidence from the semiconductor industry suggests that competing firms often choose to conduct R&D in similar areas, even when there are feasible alternative ways to conduct research. For example, the x86 instruction set architecture (ISA) is being continuously developed by rival firms Intel and AMD since the 1990s.[2] Although other instruction sets, like those developed by Sun Microsystems, are available, Intel and AMD have so far chosen similar research strategies as far as the x86 instruction set is concerned. On the other hand, there are instances when these same firms have chosen distinct research approaches. For instance, Intel and AMD have adopted distinct approaches in the recent race to develop quad-core processors.[3]

This paper therefore addresses how firms strategically choose the extent of knowledge spillover when the choice of product characteristics affects more than just a firm's marketing (pricing) ability and its R&D activity. We build a three-stage model that combines the conventional spatial competition framework (Hotelling model) and the competitive process R&D framework. In the first stage, two ex ante identical duopolistic firms simultaneously choose the locations of their products. In the second stage, they engage in competitive (cost-reducing) R&D. In the third stage, they simultaneously announce their prices in the product market. The product location chosen by a firm represents the characteristics of its product. The distance between the two firms determines the extent of effective spillover. The position of a firm's product not only reflects the product's characteristics that cater to differing tastes, but also mirrors its technological position in technology space as mentioned by Griliches (1979) and later used in empirical analysis by Jaffe (1986). When a firm chooses an idiosyncratic product configuration, it softens price competition but restricts effective

[2]Notable extensions are Intel's MMX and SSE instructions and AMD's 64-bit extensions.

[3]While Intel couples two of its existing dual-core processors within a single package, AMD imbeds four independent cores into a single chip. Both firms have made strong public statements to make known their current approaches.

spillover; when a firm attempts to maximize spillover and soften R&D competition, it has to forfeit the benefit from differentiation and face intensified price competition. The level of equilibrium product differentiation as well as the research connectedness (or spillover) thus depends on the tension between these two conflicting effects.

The modeling framework adopted in this paper is closely related to that of Kamien and Zang (2000) and Weithaus (2005). Both have addressed the issue of firm-level choice of broad versus narrow research track. They assume that firms are Cournot competitors in the product market. This paper, however, considers price competition between differentiated products and relates firms' ability to appropriate gains from spillover to their product differentiation decision.[4]

Under plausible assumptions, this paper shows that both moderate differentiation (with moderate spillover) and maximum differentiation (with minimum spillover) could arise in equilibrium. The level of differentiation or the degree of their "connectedness" crucially depends on the (exogenous) environmental factors in the model, such as the heterogeneity of consumer demand, the cost of R&D and the characteristics of the spillover function.

In particular, our results indicate that a more heterogenous market leads to more differentiated products and greater equilibrium cost reduction, but less information flow via spillover. It can be intuited that a more heterogeneous market propels firms to further segregate the market, which decreases their R&D connectedness despite the temptation of additional appropriable R&D. A more heterogenous market therefore strengthens firms' incentive for product differentiation, while the reduced appropriability weakens the free-riding effect and allows firms to step up their individual R&D investments.

Firms' location pattern also depends on the cost parameter of their R&D activities. Our results imply that the cost parameter functions analogously to the parameter that measures consumer heterogeneity: the more costly the R&D activity, the more differentiated their products and the less the knowledge spillover. This counter-intuitive effect arises from multiple sources. Firstly, firms reduce their R&D effort when the cost increases. The downsized

[4]This paper addresses how firms strategically choose the extent of knowledge spillover by directly choosing what kinds of products to develop and how much R&D investment to make in the presence of heterogenous consumers and positive R&D costs.

R&D activity in turn reduces not only the benefit firms could gain from softened R&D competition by locating closer, but also firms' need to locate closer in order to soften the R&D competition. Secondly, more costly R&D activity also weakens a firm's willingness to let its own knowledge lead to the other. Both effects cause firms' trade-off to favor more differentiated products.

Finally, firms' location pattern depends on the sensitivity or responsiveness of spillover rate to the distance between firms' products. Firms tend to reduce product differentiation when the level of spillover is more sensitive to their distance. Intuitively speaking, when the function of spillover rate is more sensitive, a closer location pattern allows firms to realize additional information flow, which therefore strengthens firms' incentive to locate closer.

The impact of environmental factors will be further elaborated upon in Sections 4 and 5. Besides providing a theoretical contribution, these insights allow for the derivation of interesting practical implications. Section 6 furthers the discussion on the divergence between equilibrium behavior and the social optimum under differing circumstances. Numerical exercises are conducted to discuss how varying environmental factors could affect the gap between equilibrium and the social optimum. The results thus shed light on the ramifications of differing R&D or market regulation policies.

2 Model

Consider a duopolistic market with two *ex-ante* identical firms, indexed by $i = 1, 2$. A "linear city" lies on a line of unit length. Firm 1 and 2 are located on the unit line at point a_1, and $1 - a_2$ respectively. Without loss of generality, assume $0 \leq a_1 \leq 1 - a_2 \leq 1$. The location of each firm metaphorically represents the characteristic of its product. A unit mass of consumers is uniformly distributed along this linear city. The type of consumer is represented by the physical location $x \in [0, 1]$, which indicates the most preferred product configuration. A consumer located at x receives a utility of $V - t(x - y)^2$ if she purchases the product located at a point y, where $t > 0$ literally measures the consumer's travel cost, and is a proxy of the degree of consumer heterogeneity.

Assume that both firms are initially endowed with the same production technology, which allows them to produce the product at a constant marginal cost $C < V$. However, firms can invest in process innovation to reduce their production costs. Firms are assumed to be equally capable of R&D in the sense that equal R&D input will result in equal R&D output. x_i units of R&D input cost $k_i = \phi x_i^2$, with $\phi > 0$, and leads to one unit of reduction in firm i's marginal cost.

Following Kamien and Zang (1988), as a result of the knowledge spillover between the two firms, a portion $\beta \in [\underline{\beta}, 1)$ of firm j's R&D input contributes to its competitor i's effective R&D. Thus, firm i's effective R&D input becomes $x_i + \beta x_j$. As a result, firm i's marginal cost of production is given by $C_i = C - x_i - \beta x_j$. Similar specifications apply for firm j. The parameter β acts as the spillover parameter that indicates the level of "leakage" or appropriability.

The novel feature of this model is that the spillover parameter β is related to firms' product locations (product configurations or characteristics). It is assumed that the greater the distance between the two firms, i.e. the more differentiated firms' products, the less the knowledge spillover can be realized by these firms. Firms integrate their choices of product design (in terms of product characteristics) with their choice of research tracks. When firms produce more similar products, they are able to realize additional spillover from each other's R&D activities. Thus, β strictly decreases with the distance between a_1 and $1 - a_2$. For the sake of tractability, in the subsequent analysis it is assumed that β linearly increase as firms get closer. Define $m \equiv 1 - a_1 - a_2$. That is, $\beta = \underline{\beta} + \gamma(1 - m)$, which is equivalent to $\beta = \underline{\beta} + \gamma(a_1 + a_2)$, with $\gamma < (1 - \underline{\beta})$.[5] Here the parameter γ measures the sensitivity of the spillover rate to firms' location pattern.

The games proceeds in three stages, and it is assumed that firms take their actions simultaneously in each stage. In the first stage, firms choose their product location, a_1 and $1 - a_2$. In the second stage, upon observing their competitors' configurations, firm i chooses its R&D input x_i. In the third stage, firms compete in the product market and simultaneously announce their prices, p_1 and p_2, respectively.

[5] A more general setup would postulate β as a function g of the distance between the firms (or equally, the location profile (a_1, a_2)), i.e., $\beta \equiv g(m)$, which strictly decreases in its argument. Numerical simulation is conducted in a later section where it is assumed that β is a concave function of m.

3 Analysis

In this section, we first derive firms' payoff functions for any given location profile (a_1, a_2), and then solve for the subgame perfect equilibrium of this three-stage game.

For the sake of analytical tractability, two simplifying regularity assumptions are made throughout this analysis.

Assumption 1 $6\phi t > (1 - \underline{\beta})^2$.

This assumption requires that the R&D activity is sufficiently costly, and/or consumer preference is sufficiently heterogenous. This assumption is crucial to guarantee the existence of sensible equilibrium. The significance and implications of this assumption will be further elaborated upon at a later point. Assumption 1 implies that a unique $\kappa \in (0, 1)$ exists that solves the following equation.

$$6\phi t(1 - \kappa) = [1 - \underline{\beta} - \gamma\kappa]^2. \tag{1}$$

The next assumption is then made on the strategy space of each firm.

Assumption 2 $a_1, a_2 \in [0, \frac{\kappa}{2}]$.

Assumption 2 imposes an upper bound on a firm's location choice towards the center, and keeps the distance between the firms bounded from below. It thus implies that minimal differentiation remains regardless, which could be interpreted as a "natural" difference between "brands". As a result, the condition $6\phi tm > (1 - \underline{\beta})^2$ will be maintained.

3.1 Preliminaries

The game is solved backward. In the last stage, given each firm's production location (a_1, a_2) and marginal cost (C_1, C_2), firm i charges a price p_i, and engages in a price competition.

This paper focuses on pure-strategy **interior** equilibrium throughout, with both firms serving positive market shares, i.e., $s_1, s_2 < 1$ with s_i denotes firm i's market share. Such an equilibrium requires that no excessive cost differential exists between the firms.

Lemma 1 *(Ziss, 1993) (i)* $s_1 = 1$ *if and only if* $C_2 - C_1 \geq tm(3 - a_1 + a_2)$. *Firm 1 receives a profit from the product market* $C_2 - C_1 - tm(1 - a_1 + a_2)$*; (ii)* $s_2 = 1$ *if and only if*

$C_1 - C_2 \geq tm(3 - a_2 + a_1)$.

When firms' cost differential remains moderate, it can then be obtained by standard technique that in an interior equilibrium, the market shares secured by firms 1 and 2, respectively, are given by

$$s_1 = a + \frac{m}{2} + \frac{p_2 - p_1}{2tm}, \tag{2}$$

and

$$s_2 = 1 - s_1 = b + \frac{m}{2} + \frac{p_1 - p_2}{2tm}, \tag{3}$$

where $m \equiv 1 - a_1 - a_2$ is the distance between the two firms' locations.

Thus, in the third stage, firm i faces the following maximization problem:

$$\max_{p_i} \pi_i = (p_i - C_i)s_i - k_i, \tag{4}$$

where s_i is given by equations (2) and (3) and k_i denotes a firm i's R&D expense.

In the first stage, Firms 1 and 2 face the following maximization problems:

$$\max_{a} \pi_1 \text{ and } \max_{b} \pi_2, \text{ respectively.} \tag{5}$$

We now attempt to find out firms' equilibrium payoffs in any given subgame with a fixed location pattern (a_1, a_2). We first sketch the main properties of such equilibrium, assuming its existence.

Lemma 2 *Assume there exists a pure-strategy interior subgame perfect equilibrium. In such an equilibrium, for a given location pair (a_1, a_2), a firm i's R&D investment is given by*

$$x_i = \frac{(1-\beta)}{18\phi}[(3 + a_i - a_j) + \frac{(1-\beta)^2(a_i - a_j)}{9\phi tm - (1-\beta)^2}], i \neq j. \tag{6}$$

At the beginning of the second stage, firm i receives an overall profit of

$$\pi_i = \frac{1}{18}[tm - \frac{(1-\beta)^2}{18\phi}][(3 + a_i - a_j) + \frac{(1-\beta)^2(a_i - a_j)}{9\phi tm - (1-\beta)^2}]^2, i \neq j. \tag{7}$$

Proof. We first derive firms' prices and R&D expenses assuming an interior equilibrium. By standard technique, we obtain that given C_1 and C_2, in an interior equilibrium firm 1 and firm 2 charge the following prices respectively:

$$p_1 = \frac{2tm(2a_1 + a_2)}{3} + tm^2 + \frac{2C_1 + C_2}{3}, \quad \text{and} \tag{8}$$

$$p_2 = \frac{2tm(a_1 + 2a_2)}{3} + tm^2 + \frac{C_1 + 2C_2}{3}, \quad \text{respectively,} \tag{9}$$

when $C_2 - C_1 < tm(3 - a_1 + a_2)$ and $C_1 - C_2 < tm(3 - a_2 + a_1)$. Thus, assuming that both firms remain active in product market, given (a_1, a_2) and (x_1, x_2), the profits for the two firms in the beginning of the third stage are given by

$$\pi_1 = \frac{[tm(3 + a_1 - a_2) - (C_1 - C_2)]^2}{18tm} - k_1, \tag{10}$$

$$\pi_2 = \frac{[tm(3 + a_2 - a_1) - (C_2 - C_1)]^2}{18tm} - k_2, \quad \text{respectively,} \tag{11}$$

where $k_i = \phi x_i^2$.

We now go to the second stage. In the second stage, given firms' production location profile (a_1, a_2), each firm i faces the following maximization problem:

$$\max_{x_i} \pi_i, \quad \text{where} \quad \pi_i \quad \text{is given by (10) and (11).}$$

When firm 1 and firm 2 invest x_1 and x_2 respectively, we have

$$\begin{aligned} C_1 - C_2 &= (C - x_1 - \beta x_2) - (C - x_2 - \beta x_1) \\ &= -(1 - \beta)(x_1 - x_2). \end{aligned} \tag{12}$$

We now assume an interior equilibrium indeed exists. Such an interior equilibrium requires

$$\frac{d\pi_1}{dx_1} = \frac{d\frac{[tm(3+a-b)+(1-\beta)(x_1-x_2)]^2}{18tm}}{dx_i} - 2\phi x_1 = 0, \tag{13}$$

$$\frac{d\pi_2}{dx_2} = \frac{d\frac{[tm(3+b-a)+(1-\beta)(x_2-x_1)]^2}{18tm}}{dx_i} - 2\phi x_2 = 0. \tag{14}$$

We then algebraically solve for the equilibrium R&D as the following:

$$x_1 = \frac{(1-\beta)}{18\phi}[(3 + a_1 - a_2) + \frac{(1-\beta)^2(a_1 - a_2)}{9\phi tm - (1-\beta)^2}] \quad \text{and} \tag{15}$$

$$x_2 = \frac{(1-\beta)}{18\phi}[(3 + a_2 - a_1) + \frac{(1-\beta)^2(a_2 - a_1)}{9\phi tm - (1-\beta)^2}], \quad \text{respectively.} \tag{16}$$

To guarantee that the solutions we give above indeed appear in an interior equilibrium, we have to verify $x_1, x_2 \geq 0$, and verify $C_2 - C_1 < tm(3 - a_1 + a_2)$ and $C_1 - C_2 < tm(3 - a_2 + a_1)$. These requirements boil down to

$$\frac{(1-\beta)^2}{9\phi}[(a_1 - a_2) + \frac{(1-\beta)^2(a_1 - a_2)}{9\phi tm - (1-\beta)^2}] \leq tm(3 - a_1 + a_2), \tag{17}$$

$$\frac{(1-\beta)^2}{9\phi}[(a_2 - a_1) + \frac{(1-\beta)^2(a_2 - a_1)}{9\phi tm - (1-\beta)^2}] \leq tm(3 + a_1 - a_2), \text{ and} \tag{18}$$

$$(3 + a_i - a_j) + \frac{(1-\beta)^2(a_i - a_j)}{9\phi tm - (1-\beta)^2} \geq 0. \tag{19}$$

The presumption of nonnegative R&D requires $3 + (a_i - a_j)\frac{9\phi tm}{9\phi tm - (1-\beta)^2} > 0$, which is guaranteed by Assumption 2. To see that, rewrite $\frac{9\phi tm}{9\phi tm - (1-\beta)^2}$ as $\frac{9\phi tm}{3\phi tm + [6\phi tm - (1-\beta)^2]}$. Because $6\phi tm - (1-\beta)^2 \geq 0$, we have $\frac{9\phi tm}{3\phi tm + [6\phi tm - (1-\beta)^2]} \leq 3$, which thus guarantee $(a_i - a_j)\frac{9\phi tm}{9\phi tm - (1-\beta)^2} > -3$.

(17) requires

$$(1-\beta)^2(a_1 - a_2) \leq [9\phi tm - (1-\beta)^2][3 - (a_1 - a_2)]. \tag{20}$$

This condition is satisfied whenever Assumption 2 holds. Because $6\phi tm \geq (1-\beta)^2$, we have LHS$\geq \frac{1}{2}(1-\beta)^2$. Hence, we only need $2(a_1 - a_2) \leq 3 - (a_1 - a_2)] \Leftrightarrow 3(a_1 - a_2) \leq 3$, which always holds.

(18) requires

$$(1-\beta)^2(a_2 - a_1) \leq [9\phi tm - (1-\beta)^2][3 - (a_2 - a_1)], \tag{21}$$

and a similar argument applies.

Q.E.D. ■

The above exercise has shown that a firm would choose x_i as given by Lemma 2 when it maximizes $\pi_i = \frac{[tm(3+a_i-a_j)-(C_i-C_j)]^2}{18tm} - k_1$. However, a gap remains before we can verify the existence of such an interior equilibrium. It must be shown that the solution of x_i indeed constitutes the best response to x_j, as given by (6). That is, given that the other firm invests x_j, firm i indeed maximizes this payoff function, and has no incentive to over-invest in order to drive the other firm out of the market. As our model allows firms to invest in process R&D to acquire cost advantage, a firm could engage in dominating R&D efforts in order to leapfrog its rival. However, Assumptions 1 and 2 stated above help rein in this incentive, and guarantee the existence of interior equilibria.

Lemma 3 *When Assumption 1 and Assumption 2 hold, the solution of x_is given by are the best responses to each other.*

Proof. As shown by Ziss (1993), firm 1 supplies to the whole market if $C_2 - C_1 \geq tm(3 - a_1 + a_2)$, and makes a profit $C_2 - C_1 - tm(1 - a_1 + a_2)$. It remains to check given that the other firm invests x_j, if it is the best response for this firm to invest x_i as the firm could increase its R&D intensity to create substantial cost advantage and drives the other firm out of the firm.

Now consider firm one indeed achieves this cost margin, and receives a profit $(1-\beta)(x_1 - x_2) - tm(1 - a_1 + a_2) - \phi x_1^2$. Note that the payoff of the firm is bounded by $(1-\beta)[\frac{(1-\beta)}{2\phi} - x_2] - tm(1 - a_1 + a_2) - \phi\frac{(1-\beta)^2}{4\phi}$. This upper bound is obtained by maximizing the payoff $C_2 - C_1 - tm(1 - a_1 + a_2) - k_1$ for a given x_2, while the condition $C_2 - C_1 \geq tm(3 - a_1 + a_2)$ is not checked yet. If the firm chooses x_1 as given by (6), then the firm receives the payoff as given by (7). We now claim (7) $> (1-\beta)[\frac{(1-\beta)}{2\phi} - x_2] - tm(1 - a_1 + a_2) - \phi\frac{(1-\beta)^2}{4\phi}$. That is,

$$\begin{aligned} &\frac{1}{18}[tm - \frac{(1-\beta)^2}{18\phi}][3 + (a_1 - a_2) + \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm - (1-\beta)^2}]^2 \\ > \; &\frac{(1-\beta)^2}{4\phi} - tm(1 - a_1 + a_2) - \frac{(1-\beta)^2}{18\phi}[(3 - (a_1 - a_2) - \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm - (1-\beta)^2}]. \end{aligned} \tag{22}$$

Assumption 2 implies $a_1, a_2 < \frac{1}{2}$, while Assumption 1 implies $\frac{9\phi tm}{9\phi tm-(1-\beta)^2} \leq 3$. We then obtain $(a_1 - a_2) + \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm-(1-\beta)^2} \in (-\frac{3}{2}, \frac{3}{2})$ and $tm(1 - a_1 + a_2) > \frac{tm}{2}$. We consider two cases. Furthermore, we obtain from Assumption 1 $\frac{(1-\beta)^2}{18\phi} \leq \frac{tm}{3}$, which gives $tm - \frac{(1-\beta)^2}{18\phi} \geq \frac{2tm}{3}$.

Case 1: $a_1 - a_2 \leq 0$. $\frac{(1-\beta)^2}{4\phi} - \frac{(1-\beta)^2}{18\phi}[(3 - (a_1 - a_2) - \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm-(1-\beta)^2}] \leq \frac{(1-\beta)^2}{4\phi} - \frac{(1-\beta)^2}{6\phi} = \frac{(1-\beta)^2}{12\phi} \leq \frac{tm}{2}$. Then we obtain RHS is nonpositive because $tm(1 - a_1 + a_2) \geq \frac{tm}{2}$.

Case 2: $a_1 - a_2 > 0$. Hence, $\frac{(1-\beta)^2}{4\phi} - tm(1-a_1+a_2) - \frac{(1-\beta)^2}{18\phi}[(3-(a_1-a_2) - \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm-(1-\beta)^2}] < \frac{(1-\beta)^2}{18\phi}(a_1 - a_2) \cdot \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm-(1-\beta)^2}]$. We define $d \equiv (a_1 - a_2) \cdot \frac{(1-\beta)^2(a_1-a_2)}{9\phi tm-(1-\beta)^2}$. Then we only need to show

$$\frac{tm - \frac{(1-\beta)^2}{18\phi}}{18} \cdot (3 + d)^2 > \frac{(1-\beta)^2}{18\phi} \cdot d. \tag{23}$$

We have $\frac{tm - \frac{(1-\beta)^2}{18\phi}}{18} \cdot (3+d)^2 - \frac{(1-\beta)^2}{18\phi} \cdot d = \frac{(3+d)^2}{18} \cdot tm - \frac{(3+d)^2}{18} \cdot \frac{(1-\beta)^2}{18\phi} - d \cdot \frac{(1-\beta)^2}{18\phi}$, which is rewritten as $\frac{(3+d)^2}{18} \cdot tm - \frac{(1-\beta)^2}{18\phi}[\frac{(3+d)^2}{18} + d] \geq \frac{(3+d)^2}{18} \cdot tm - \frac{tm}{3}[\frac{(3+d)^2}{18} + d]$. It is therefore equivalent to showing $d^2 - 3d + 9 > 0$, which is always satisfied because $d^2 - 3d + 3 > d^2 - 3d + \frac{9}{4} = (d - \frac{3}{2})^2 > 0$.

Hence, we conclude that when firm 2 invests x_2 as given by (9), x_1 as given by (8) is the best response to x_2. The same technique would apply to prove x_2 as the best response to x_1.

Q.E.D. ■

Lemma 3 therefore guarantees that x_is (as given by (6)) indeed constitute best responses to each other, and the payoff function of (7) is continuous, when our assumptions hold. Combining Lemma 2 and Lemma 3, the following can then be concluded.

Proposition 1 *When Assumption 1 and Assumption 2 hold, for any given location pair* (a_1, a_2)*, there exists a unique interior subgame perfect equilibrium where both firms serve positive market shares, as characterized by Lemma 2.*

The two assumptions made are critical to guarantee the existence of such interior equilibrium. Assumption 1 simply prevents each firm from engaging in excessive R&D effort that could drive the other out of the market. Firstly, as implied by (7), the payoff from coexistence could be sufficiently attractive to each firm only if the term tm is sufficiently large relative to the term $(1-\beta)^2$. Secondly, an interior equilibrium breaks down when a firm makes excessive investment to create a substantial cost differential. Thus, an interior equilibrium requires significant R&D costs, i.e., sufficiently large ϕ, to rein in a firm's incentive for excessive investment. Assumption 2 plays a more subtle role here. An interior equilibrium would not exist when firms are located excessively close to each other. The assumption enforces a minimum distance between firms. Obviously, when firms are located closer together, the term $(1-\beta)^2$ strictly increases. Consequently, the input required for any given amount of cost reduction declines due to excessive spillover. In addition, a smaller distance further exacerbates subsequent price competition because of the increasing homogeneity between the offered products. It therefore reduces the required cost advantage for a firm to drive the other out of market. Both effects would lure firms to invest dominating R&D. Hence, it becomes more difficult to retain an interior equilibrium. When firms are located excessively close, and when they are allowed to invest in cost-reducing R&D, multiple equilibria necessarily arise, and it is difficult to characterize the payoff structures in such subgame without additional assumptions. To avoid this complication and allow ourselves to focus on firms' basic trade-off between pricing ability and R&D appropriability, we assume away this possibility by restricting firms' location space.

3.2 Equilibrium

For simplicity, we focus on symmetric equilibrium with $a_1^* = a_2^* = a^*$ in the subsequent analysis. By Lemma 2, a symmetric interior equilibrium would yield an equilibrium level of R&D effort $x = \frac{(1-\beta)}{6\phi}$.

Corollary 1 *When firms choose symmetric locations, i.e., $a_1 = a_2 = a$, each firm thus receives a profit*

$$\pi_2 = \frac{1}{2}[tm - \frac{(1-\beta)^2}{18\phi}], \tag{24}$$

where $m = 1 - 2a$ and $\beta = \underline{\beta} + 2a\gamma$.

Thanks to Lemma 2 and 3, when Assumptions 1 and 2 hold, the payoff for a firm is continuous on its location and differentiable. Without loss of generality, we focus on Firm 1 in our subsequent analysis. Taking the first order derivative of π_1 with respect to a_1 yields

$$\begin{aligned} \frac{\partial \pi_1}{\partial a_1} &= \frac{1}{18}[-t + \frac{(1-\beta)}{9\phi}\gamma][(3 + a_1 - a_2) + \frac{(1-\beta)^2(a_1 - a_2)}{9\phi tm - (1-\beta)^2}]^2 \\ &+ \frac{1}{18} \cdot 2[tm - \frac{(1-\beta)^2}{18\phi}][(3 + a_1 - a_2) + \frac{(1-\beta)^2(a_1 - a_2)}{9\phi tm - (1-\beta)^2}] \\ &\{1 + \frac{[-2(a_1 - a_2)(1-\beta)\gamma + (1-\beta)^2][9\phi tm - (1-\beta)^2]}{[9\phi tm - (1-\beta)^2]^2} \\ &+ \frac{(1-\beta)^2(a_1 - a_2)[9\phi t - 2(1-\beta)\gamma]}{[9\phi tm - (1-\beta)^2]^2}\}, \end{aligned} \tag{25}$$

when $a_1 \in (0, \frac{\kappa}{2})$. Firm 1 would have an incentive to move towards the center, i.e., increasing a_1 whenever $\frac{\partial \pi_1}{\partial a_1} > 0$.

Consider $\left.\frac{\partial \pi_1}{\partial a_1}\right|_{a_1 = a_2 = a}$ as a function of the distance m and define $F(m) \equiv \left.\frac{\partial \pi_1}{\partial a_1}\right|_{a_1 = a_2 = a}$. We are interested in symmetric interior equilibrium with $a_1^* = a_2^* = a^*$. When $a_1 = a_2 = a$, $\frac{\partial \pi_1}{\partial a_1}$ is given by

$$F(m) = \left.\frac{\partial \pi_1}{\partial a_1}\right|_{a_1 = a_2} = -\frac{3}{9\phi}[9\phi t - (1-\beta)\gamma] + tm[1 + \frac{9\phi tm}{9\phi tm - (1-\beta)^2}], \tag{26}$$

where $m = 1 - 2a$.

The following Lemma sketches out one important property of the function $F(m)$.

Lemma 4 *$F(m)$ strictly increases with the distance m, for any m that satisfies $6\phi tm - (1-\beta)^2$.*

Proof. Obviously, $-\frac{3}{9\phi}[9\phi t - (1-\beta)\gamma]$ would increase as m increase, as a greater distance reduces β. Then we consider the effect of m on the term $F_2(m) = m[1 + \frac{9\phi tm}{9\phi tm-(1-\beta)^2}]$. Take first order derivative of it with respect to m, and we have

$$\begin{aligned} F_2^{'}(m) &= \frac{18\phi tm - (1-\beta)^2}{9\phi tm - (1-\beta)^2} \\ &+m\frac{[18\phi t - 2(1-\beta)\gamma][9\phi tm - (1-\beta)^2] - [18\phi tm - (1-\beta)^2][9\phi t - 2(1-\beta)\gamma]}{[9\phi tm - (1-\beta)^2]^2}. \end{aligned} \tag{27}$$

We consider three possible cases.

Case 1: $9\phi t < (1-\beta)\gamma$**.**

We consider the sign of $[18\phi t - 2(1-\beta)\gamma][9\phi tm - (1-\beta)^2] - [18\phi tm - (1-\beta)^2][9\phi t - 2(1-\beta)\gamma]$. We rewrite it as

$$\begin{aligned} & [18\phi tm - (1-\beta)^2][2(1-\beta)\gamma - 9\phi t] - 2[(1-\beta)\gamma - 9\phi t][9\phi tm - (1-\beta)^2] \\ = \ & [18\phi tm - (1-\beta)^2][2(1-\beta)\gamma - 9\phi t] - [(1-\beta)\gamma - 9\phi t][18\phi tm - 2(1-\beta)^2]. \end{aligned} \tag{28}$$

It is strictly positive because $2(1-\beta)\gamma - 9\phi t > (1-\beta)\gamma - 9\phi t > 0$, and $18\phi tm - (1-\beta)^2 > 18\phi tm - 2(1-\beta)^2 > 0$. Thus, we show $F_2^{'}(m) > 0$.

Case 2: $(1-\beta)\gamma \leq 9\phi t \leq 2(1-\beta)\gamma$.

It is straightforward to verify that $[18\phi t - 2(1-\beta)\gamma][9\phi tm - (1-\beta)^2] - [18\phi tm - (1-\beta)^2][9\phi t - 2(1-\beta)\gamma]$ is positive. Thus, we show $F_2^{'}(m) > 0$.

Case 3: $9\phi t > 2(1-\beta)\gamma$.

We first rearrange the term $[18\phi t - 2(1-\beta)\gamma][9\phi tm - (1-\beta)^2] - [18\phi tm - (1-\beta)^2][9\phi t - 2(1-\beta)\gamma]$, and we obtain

$$\begin{aligned} & [18\phi t - 2(1-\beta)\gamma][9\phi tm - (1-\beta)^2] - [18\phi tm - (1-\beta)^2][9\phi t - 2(1-\beta)\gamma] \\ = \ & 18\phi tm \cdot 2(1-\beta)\gamma + 9\phi t(1-\beta)^2 - 18\phi t(1-\beta)^2 - 2(1-\beta)\gamma \cdot 9\phi tm \\ = \ & 9\phi tm \cdot 2(1-\beta)\gamma - 9\phi t(1-\beta)^2. \end{aligned} \tag{29}$$

Insert (29) into (27), and we have

$$
\begin{aligned}
F_2'(m) &= \frac{18\phi tm - (1-\beta)^2}{9\phi tm - (1-\beta)^2} + m\frac{[9\phi tm \cdot 2(1-\beta)\gamma - 9\phi t(1-\beta)^2]}{9\phi tm - (1-\beta)^2} \\
&= \frac{[18\phi tm - (1-\beta)^2] + 9\phi tm[2(1-\beta)\gamma - (1-\beta)^2]}{9\phi tm - (1-\beta)^2}, \qquad (30)
\end{aligned}
$$

which is obvious positive because $18\phi tm - (1-\beta)^2 \geq 12\phi tm$, and $12\phi tm > 9\phi tm(1-\beta)^2$.

Q.E.D. ■

This shows that $F(m)$ is continuous and strictly increasing. Now the symmetric subgame perfect equilibrium of this game is ready to be derived.

In any symmetric equilibrium with $a_1^* = a_2^* = a^*$ and $m^* = 1 - 2a^*$, the following must hold: $m^* = 1$ if $F(0) \leq 0$, $m^* > 1$ if $F(0) > 0$. When $F(1-\kappa) \geq 0$, we then have $m^* = 1-\kappa$.

Proposition 2 *A symmetric equilibrium that involves moderate differentiation, i.e.,* $a_1^* = a_2^* = a^* \in (0, \frac{\kappa}{2})$, *exists if the following holds:*

(1) $\gamma \geq \frac{3\phi t}{(1-\underline{\beta})}[3 - \frac{18\phi t-(1-\underline{\beta})^2}{9\phi t-(1-\underline{\beta})^2}]$;*and (2)* $9\phi t < \frac{15+\sqrt{117}}{6}(1-\underline{\beta})^2$.

Proof. To have an equilibrium involving moderate differentiation, it requires $F(1) > 0$. To make that, we need

$$
\begin{aligned}
\frac{3\phi t[18\phi t - (1-\underline{\beta})^2]}{9\phi t - (1-\underline{\beta})^2} &> 9\phi t - (1-\underline{\beta})\gamma \Rightarrow \\
\gamma &\geq \gamma^* = \frac{3\phi t}{(1-\underline{\beta})}[3 - \frac{18\phi t - (1-\underline{\beta})^2}{9\phi t - (1-\underline{\beta})^2}]. \qquad (31)
\end{aligned}
$$

Because $\gamma < (1-\underline{\beta})$, thus, the γ^* could exist only if $\frac{15-\sqrt{117}}{6}(\approx 0.7)(1-\underline{\beta})^2 < 9\phi t < \frac{15+\sqrt{117}}{6}(\approx 4.3)(1-\underline{\beta})^2$. By the assumption $\gamma < (1-\underline{\beta})$, we only need $9\phi t < \frac{15+\sqrt{117}}{6}(1-\underline{\beta})^2$.

By Lemma 4, if $F(1-\kappa) < 0$, we can always locate $a^* < \frac{\kappa}{2}$ such that $\frac{\partial \pi_1}{\partial a_1}\Big|_{a_1=a_2=a^*} = 0$. If $F(1-\kappa) \geq 0$, then we conclude that $\frac{\partial \pi_1}{\partial a_1}\Big|_{a_1=a_2=a^*} > 0$ for all $a < \frac{\kappa}{2}$. We therefore end up with $a^* = \frac{\kappa}{2}$.

Q.E.D. ■

As implied by the proof of Proposition 2, an equilibrium distance $m^* \in (0, 1-\kappa)$ would emerge in the equilibrium if $2(1-\beta)^2 + (1-\beta)\gamma < 9\phi t$, for $a = \frac{\kappa}{2}$. In this case, the equilibrium distance between the firms would be obtained by equalizing

$$
[9\phi t - (1-\beta)\gamma][9\phi tm^* - (1-\beta)^2] = 3\phi tm^*[18\phi tm^* - (1-\beta)^2], \qquad (32)
$$

where a unique positive solution would apply.

Proposition 2 thus shows the existence of symmetric interior subgame perfect equilibrium of this game that involves moderate differentiation. Such equilibrium location pattern thus departs from the prediction of standard Hotelling model. The above results are intuitive. For a given set of consumers, only if R&D investment is sufficiently costly, firms would choose the product location such that they obtain some positive spillover at the cost of product differentiation. A more detailed interpretation is provided at a later point.

The proof of Proposition 2 and Lemma 4 directly imply the following.

Proposition 3 *A symmetric equilibrium that involves maximal differentiation, i.e., $a_1^* = a_2^* = 0$, exists if either condition (1) or condition (2) in Proposition 2 does not hold.*

When either of the two conditions stated by Proposition 2 is not satisfied, a firm would always prefer to locate its product as far away as possible from that of the other firm. An equilibrium is therefore obtained that results in maximal differentiation, which coincides with the prediction of the standard Hotelling model.

4 Discussion

This section investigates the underlying forces that drive our results. The primary focus of this paper is on firms' strategic choices of the "locations" of their products. Firms' location patterns affect their payoffs through two venues. It directly alters firms' incentive to conduct R&D as well as the resultant cost reduction; while it further affects firms' pricing behaviors in the product market.

Firms' (symmetric) locations are fixed, i.e., assume $a_1 = a_2 = a$. The impact of the (symmetric) location on firms' behavior is then observed in the subsequent subgame. This is begun by examining its impact on firms' R&D activity. Each firm chooses the symmetric R&D effort, $x = \frac{(1-\beta)}{6\phi}$, which costs $k = \phi x^2 = \frac{(1-\beta)^2}{36\phi}$. Each firm, therefore, realizes a cost reduction of $\Delta C = (1+\beta)x = \frac{(1-\beta^2)}{6\phi}$. The following is directly obtained:

Corollary 2 *When firms choose a symmetric location with $a_1 = a_2 = a$, the subsequent equilibrium R&D effort and equilibrium cost reduction strictly increase with the distance between them, $m = 1 - 2a$.*

When firms locate further away from each other, the knowledge spillover rate falls, which weakens firms' incentive to free-ride on each other, and increases the marginal payoff of additional R&D effort. Consequently, R&D competition escalates in the subsequent subgame.

Next, the impact of distance on prevailing market price is analyzed. In a subgame with symmetric location $a_1 = a_2 = a$, we have

$$p = 2t(1-2a)a + t(1-2a)^2 + [C - \frac{(1-\beta^2)}{6\phi}]. \tag{33}$$

When firms are located closer together, two conflicting effects come into play. Lesser differentiation forces them to charge a lower price, while diluted R&D competition leads to less cost reduction, which restrict firms' capability to reduce their prices. Taking the first order derivative with respect to a yields

$$\begin{aligned}\frac{dp}{da} &= 2t(1-4a) - 4t(1-2a) + \frac{4(1-\beta)\gamma}{6\phi} \\ &= -\frac{1}{6\phi}[12\phi t - 4(1-\beta)\gamma] < 0. \end{aligned} \tag{34}$$

The negative sign of $\frac{dp}{da}$ confirms that the former effect dominates the latter. However, both lower price and higher cost adversely affect a firm's profitability. Corollary 3 follows immediately.

Corollary 3 *Given a symmetric location pair (a, a), each firm's markup $p - C_i$ strictly decreases when firms locate closer to each other.*

The above results clearly indicate the two conflicting effects on firms' location choice. A closer distance between firms escalates price competition, which shrinks the profit margin. On the other hand, close proximity softens R&D competition and reduces R&D expenditure. Firms thus have to choose their locations in such a way as to strike a balance between these two effects. The exact opposite effects are observed when firms locate further away from each other.

The fundamental trade-off can be further witnessed by taking a closer look at a firm's (symmetric) equilibrium payoff

$$\pi(m) = \frac{1}{2}[tm - \frac{(1-\beta)^2}{18\phi}]. \tag{35}$$

The profit $\pi(m)$ can be broken up into two components, i.e, tm and $-\frac{(1-\beta)^2}{18\phi}$, but these components move in opposite directions. The first component represents the benefit a firm could reap from product differentiation, as greater differentiation softens price competition. The second component represents the benefit that arises from softened R&D competition, where a closer location pattern increases spillover and softens R&D competition.

Despite that an interior equilibrium demands the former dominate the latter (see our discussion in 3.1 regarding the roles played by our assumptions), it is shown that $6\phi t < \frac{15+\sqrt{117}}{6}(1-\underline{\beta})^2$ is required for the existence of equilibria involving moderate differentiation. This condition implies that moderate differentiation requires the sizes of ϕt be bound from above. t is the measure of demand heterogeneity. Intuitively, a more heterogeneous market creates additional room for further differentiated products. As a result, when t is excessively large, the benefit from differentiation (softened price competition) would outweigh the loss from less R&D competition. Thus, t is required to remain in a medium range for the moderately differentiated equilibrium to exist.

In addition, it comes as no surprise that moderate differentiation ($a > 0$) requires γ to be sufficiently large, i.e., $\gamma \geq \frac{3\phi t}{(1-\underline{\beta})}[3 - \frac{18\phi t-(1-\underline{\beta})^2}{9\phi t-(1-\underline{\beta})^2}]$. The parameter γ measures the responsiveness of the spillover rate to firms' distance. It also indicates a firm's marginal gain of spillover. A firm is willing to forgo the benefits gained from reduced price competition only if significant benefits can be reaped from additional spillover.

The R&D cost parameter, ϕ, negatively impacts the R&D spillover. A larger value ϕ tends to favor more differentiated products. It may at first seem counterintuitive to observe that ϕ plays a role analogous to that of t. One would imagine that when research is more costly the gain from spillover would increase, which will consequently favor less differentiation.

The following is the logic that underlies this observation. When the size of ϕ increases, it dampens firms' incentive to engage in R&D activity, and it weakens firms' incentive to engage in competitive R&D. As a result, a greater cost parameter ϕ plays an analogous role to that of a closer location pattern, as it tends to reduce R&D investments in any equilibrium. On the one hand, it substitutes away a firm's need to move towards the center (in order to reduce R&D competition): It softens R&D competition as well, while it does not escalates price competition. On the other hand, reduced R&D activities decrease the gain from spillover, which also weakens a firm's incentive to locater closer to its rival. In addition, as a given level of R&D effort is now more costly for a firm, each firm will be reluctant to allow the other to gain benefits from the spillover. If ϕ is large enough, then in the symmetric equilibrium each firm will tend to restrict the other firm's access to spillover by locating further away.

These intuitions will be further illustrated in the numerical analysis presented below.

5 Numerical Exercise and Comparative Statistics

We continue the discussion of the impact of the parameters on firms' equilibrium behavior by exercising numerical exercises. We assumed that $\beta = \underline{\beta} + \gamma(1 - m)$, with $0 < \gamma < 1$ and $0 \leq \beta < 1$. This exercise attempts to ascertain what sort of impact different external conditions (represented by the parameters) have on the various decision variables of the firm.

We impose those necessary conditions to ensure that the symmetric equilibrium exists. The following parameter values are used for the benchmark case: $\underline{\beta} = 0.15$, $\gamma = 0.65$, $t = 0.5$, $\phi = 0.4$. The impact of the change in the parameters of this model on location choice (m), R&D expenditure (x), cost reduction (ΔC) and price margin ($P - C$) need to be ascertained. The three parameters considered, ϕ, t and γ, provide three sets of results.

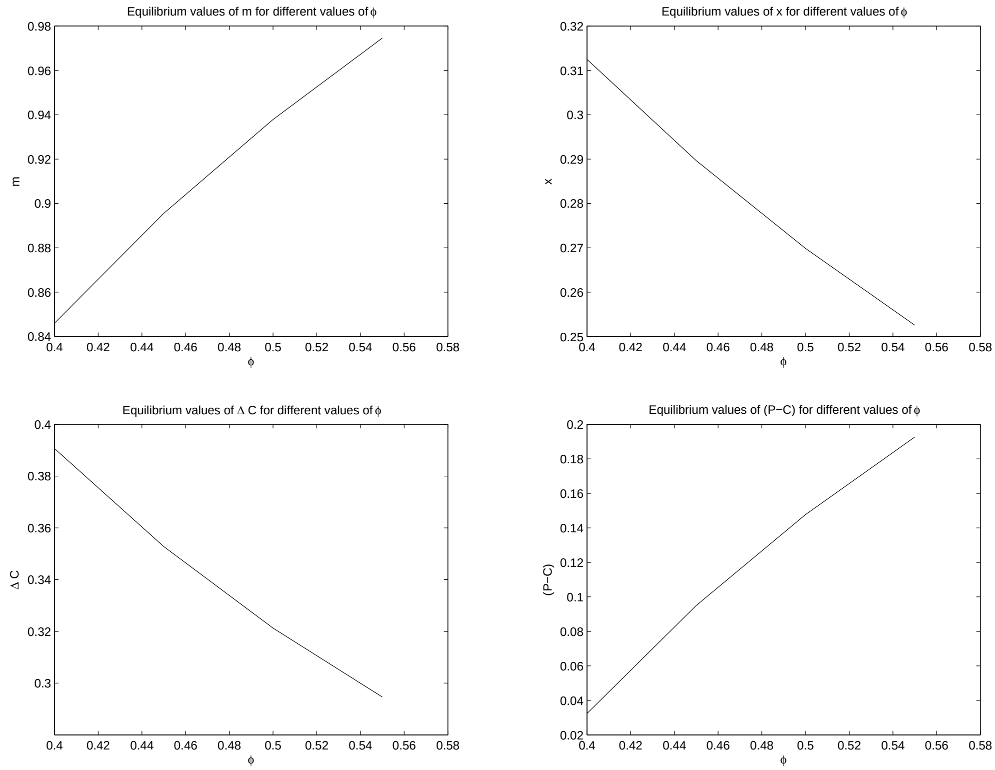

Figure 1: Equilibrium m, x, ΔC and $(P - C)$ when ϕ changes

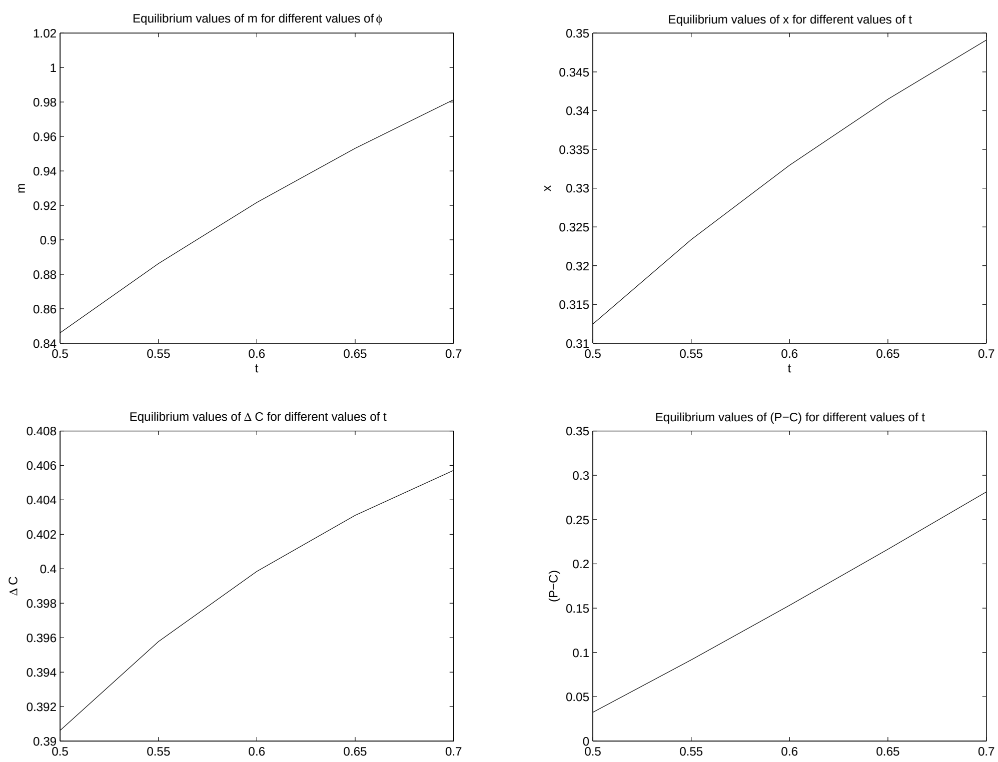

Figure 2: Equilibrium m, x, ΔC and $(P - C)$ when t changes

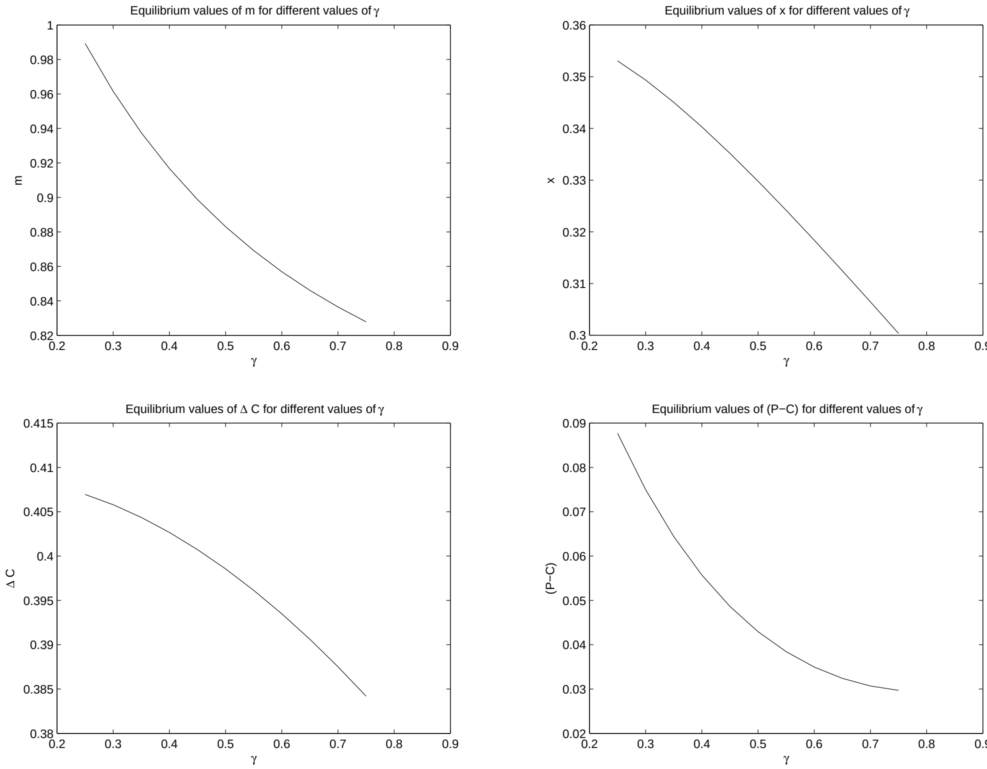

Figure 3: Equilibrium m, x, ΔC and $(P - C)$ when γ changes

The effect of an increasing ϕ (research cost) on the above-mentioned variables is first observed. It is expected that, with a higher R&D cost, the products of the firms are increasingly differentiated. As argued above, a higher value of ϕ weakens a firm's incentive to engage in excessive R&D competition, which in turn amplifies the benefit from softened price competition.

The four graphs in Figure 1 provide the first set of results. They illustrate that more costly research forces firms to choose more differentiated products and locate further away from each other.[6] Higher research costs also force firms to choose lower levels of R&D expenditures, which in turn lowers the cost reduction. However, the associated higher product differentiation ensures that the firms can charge a higher price. A high cost of research is, therefore, welfare-reducing in the sense that it decreases firms' cost reduction, while at the same time increases the price for the consumers.[7]

Next, the effect of increasing t (consumer heterogeneity) is observed. The results are

[6]For the symmetric equilibrium to exist, ϕ must be greater than 0.4.

[7]The social planner's problem is also explicitly considered in the next section.

illustrated in the four charts in Figure 2. As discussed in the previous section, the parameter t plays a role similar to that of ϕ in determining equilibrium distance. The graphs show that as consumers become more and more heterogenous in symmetric equilibrium, firms move further away from each other.[8] Higher consumer heterogeneity has a positive impact on firms' R&D expenditure as firms increase their R&D to develop differentiated products, for which they charge higher prices. Higher R&D also yields additional cost reduction.

The four charts in Figure 3 illustrate the effect of increasing γ (the sensitivity of spillover rate to firms' location distance). As γ increases in symmetric equilibrium, firms move closer to each other. It is intuitive because a larger γ allows firms to realize more spillover by locating closer to each other.[9] Since a higher value of γ reduces product differentiation, it also leads to firms choosing lower levels of R&D expenditure and ending up with a lower profit margin.

In summarizing the simulation exercise, it can be noted that in symmetric equilibrium, the parameters ϕ and t have similar impacts on the equilibrium level of product differentiation, while the impact of γ is different. A higher value of either ϕ or t tends to push the firms apart in terms of product characteristics, while a higher value of γ tends to move the firms closer together.

However, it can also be observed that the parameters ϕ and t exert remarkably different impacts on firms' equilibrium R&D effort and cost reduction. A lower value of t depresses firms' R&D effort and increases the cost. The equilibrium price margin shrinks because firms choose to produce less differentiated products. In contrast, a lower value of ϕ increases R&D while decreasing price. A lower ϕ affects firms' incentive to engage in R&D effort through two avenues. Firms increase R&D because of the lower cost, which represents a positive **direct** effect; while firms tend to decrease R&D because a smaller ϕ prompt firms to move closer to each other, which represents a negative **indirect** effect. However, the direct effect dominates the indirect effect, which leads to additional cost reduction.

It should be noted that the above results would hold even when β is non-linear.[10]

[8] For the symmetric equilibrium to exist, t must be greater than 0.5.

[9] In order for the symmetric equilibrium to exist, ϕ must be greater than 0.3.

[10] The entire exercise has been re-worked using the following functional form: $\beta(m) \equiv \overline{\beta} + (1-m)^{\alpha}(\overline{\beta} - \underline{\beta})$,

Social Welfare

In this section, the welfare problem is presented and analyzed. When firms move towards the center of the linear city, the move is expected to have mixed effects on social welfare. In a standard Hotelling model, evaluating social welfare only has to measure "travel cost" incurred on consumers (e.g., consumers' cost of adapting to products that do not fit well into their preferences). By way of contrast, our model has to take into account the effect of location pattern on the net gain from cost reduction.

We assume that firms are symmetrically located with $a_1 = a_2 = a$. The total travel cost for all consumers is given by

$$\begin{aligned} \Pi_1(a) &= 2\int_0^{1/2} t(a-x)^2 dx \\ &= \frac{2t}{3}[a^3 - (a-\frac{1}{2})^3]. \end{aligned} \tag{36}$$

Social welfare is enhanced by minimizing consumers' travel costs. It is well known in the literature that without restriction on firms' action space, $\Pi_1(a)$ would be minimized when $a = \frac{1}{4}$, i.e., when the distance between the two firms is $\frac{1}{2}$.

On the other hand, R&D achieves cost reduction and the net gain is given by

$$\begin{aligned} \Pi_2(a) &= \Delta C - 2k \\ &= \frac{(1-\underline{\beta}-2a\gamma)(1+2\underline{\beta}+4a\gamma)}{9\phi}, \end{aligned} \tag{37}$$

where k is the symmetric R&D expense. Social welfare will be enhanced by increasing this net gain.

The total social surplus is then given by: $\Pi(a) = -\Pi_1(a) + \Pi_2(a)$. $\Pi(a)$ is maximized to obtain the (symmetric) optimal firm location. The first order derivative of $\Pi(a)$ with respect to a yields

$$\frac{d\Pi(a)}{da} = -t(2a - \frac{1}{2}) + \frac{2\gamma[1-4\underline{\beta}-8a\gamma]}{9\phi}. \tag{38}$$

This gives a closed-form solution for a in terms of the parameters of the model

$$a^o = \max(\min(\frac{9\phi t + 4\gamma(1-4\underline{\beta})}{36\phi t + 32\gamma^2}, \frac{\kappa}{2}), 0), \tag{39}$$

with $0 < \underline{\beta} < \overline{\beta} < 1$ and $0 < \alpha < 1$ and similar results were obtained.

where the superscript o stands for the optimal value of the variable. In particular, when $9\phi t + 4\gamma(1 - 4\underline{\beta}) < 0$, the social optimum requires firms to engage in maximal differentiation.

The graphs below compare the optimal distance between firms (m^O) to the equilibrium distance between firms (m^*) for different values of the parameters.[11] The range of parameters considered here ensures the existence of interior equilibrium.

Figure 4 shows the effect of an increasing ϕ (research cost) on m^O (socially optimal distance) and m^* (equilibrium distance). The previous section has discussed why equilibrium product differentiation increases with the research cost parameter ϕ. This graph additionally shows that the social optimum requires that product differentiation fall with increasing research costs. It is intuitive that higher research costs cause society to prefer a closer location pattern in order to gain more from spillover by reducing excessive R&D expenditure. As a result, when research costs increase, the gap between equilibrium differentiation and social optimum widens, implying that the firms choose to differentiate their products even more and move further away from the optimal level of differentiation.

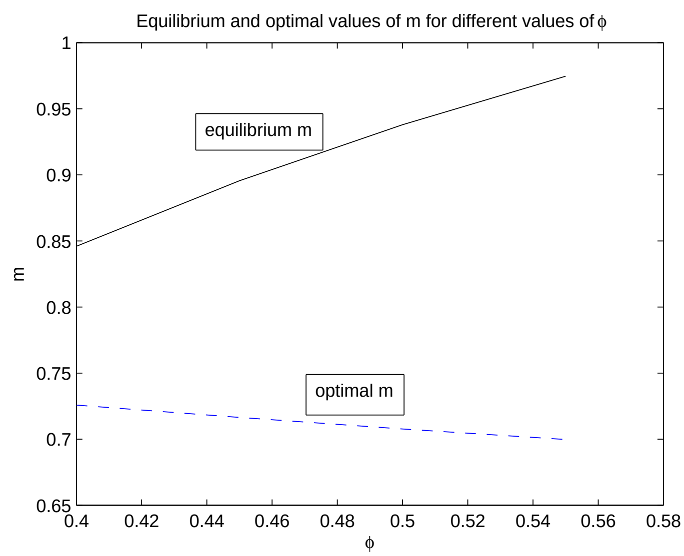

Figure 4: Equilibrium and optimal values of m for different values of ϕ

Similar observations can be made from Figure 5, which shows the effect of an increasing t (consumer heterogeneity) on m^O and m^*. It has been shown before that the parameters ϕ and

[11]While the optimal value is calculated from the analytical solution obtained above, the equilibrium value is obtained numerically.

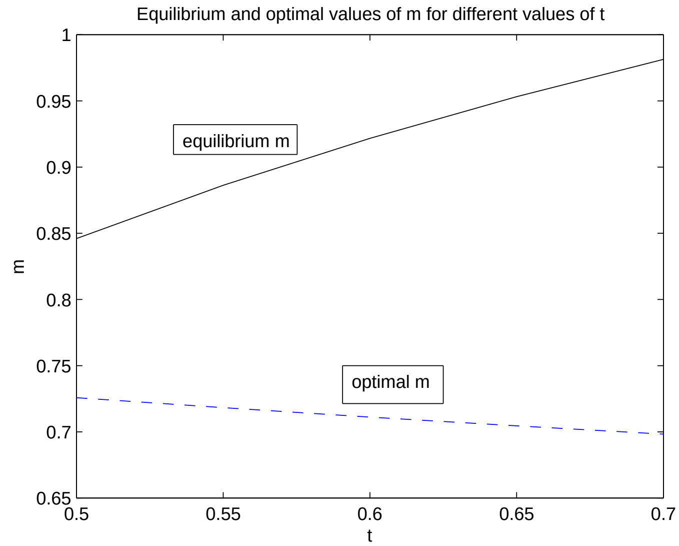

Figure 5: Equilibrium and optimal values of m for different values of t

t have a similar impact on equilibrium product differentiation. It can also be seen that the social optimum requires that product differentiation falls with an increasing level of consumer heterogeneity. Intuitively, a more heterogeneous market lures the firms to differentiate more in order to soften the price competition, which runs in contrast to the interests of social planner: Additional product differentiation increases the consumers' "travel cost" on the one hand, while it reduces the extent of knowledge spillover on the other.

Figure 6 shows the effect of changing γ on m^O and m^*. An increase in γ increases the marginal gain each firm could receive as spillover. In symmetric equilibrium, firms would therefore prefer to move closer in order to soften the R&D competition rather than softening the price competition in the product market. The numerical exercise demonstrates that the socially optimal level of product differentiation increases with the parameter γ, which in turn narrows the gap between m^O and m^*. This observation may seem counterintuitive. One would expect that as the marginal gain of spillover is amplified, it will be in society's best interest to have less differentiated products. Nevertheless, a competing force emerges: as γ increases, obtaining a certain desired level (from society's viewpoint) of spillover requires a lesser degree of product differentiation. This effect thus allows the society to increase differentiation (which could reduce consumers' travel costs and increase cost reduction) but not at the cost of spillover. Figure 6 shows that firms consistently undertake excessive

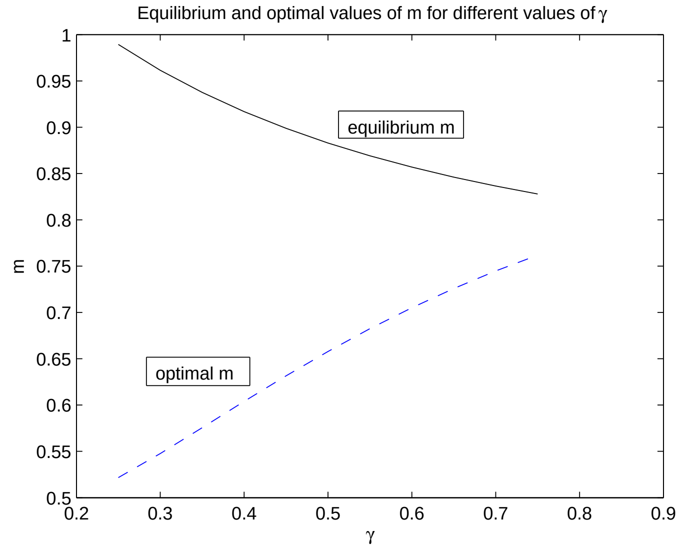

Figure 6: Equilibrium and optimal values of m for different values of γ

differentiation. An increasing γ thus better aligns firms' interests with the society. The gap between equilibrium differentiation and optimal differentiation therefore shrinks.

All the above graphs show that, for the relevant range of parameter values, firms conduct excessive differentiation in equilibrium as compared to the social optimum. That is, firms always choose more differentiation than what is socially optimal. This simply shows that, for the range of parameters for which moderate equilibrium exists, firms value the knowledge spillover less than the society as a whole. This is hardly surprising given the positive externality created by knowledge spillover, not all of which can be appropriated by the firms.

5.1 Policy Implications

These observations yield interesting policy implications. An increase in research costs results in firms choosing to differentiate their products more than what is optimal and the gap increases as research costs (ϕ) keep on increasing. Excessive product differentiation reduces the amount of benefit firms could realize through knowledge spillover, which in turn leads to excessive duplicated R&D expenditure and higher product prices. It comes as no surprise that it is in the planner's best interest to reduce firms' R&D costs.

Consequently, a policy that reduces R&D costs could benefit not only firms but also consumers. Direct measures, such as publicly funded research, can be used to bring down the cost of private research activities. For example, the Ministry of Economic Affairs in Taiwan has set up the Industrial Technology Research Institute (ITRI), which provides "technological, research, and consulting services" to domestic firms. ITRI has provided valuable services for the development and success of Taiwan's Hsinchu Science Park, a highly successful semiconductor manufacturing hub. Alternatively, allowing firms to access the knowledge from publicly funded basic research could also increase firms' knowledge stock and reduce the costs of their autonomous R&D efforts.

As the numerical exercises conducted in this research exhibit, an increasing γ could narrow the gap between equilibrium product differentiation and social optimum. This observation suggests that a policy that increases spillover would reduce excessive differentiation and improve social welfare. An increase in γ can be possibly achieved by implementing a more liberal patent policy, which allows firms to appropriate additional knowledge produced by others. Alternatively, the government could create such an environment by encouraging firms to share their knowledge. For example, the U.S. based non-profit organization, SEMATECH, encourages membership from semiconductor firms and functions as a network to help members to deal with "high costs of advanced research", "technological limitations of materials and processes" and "high costs of manufacturing".

The above exercise also shows that an increase in consumer heterogeneity (t) causes firms to choose more differentiated products, consequently causing the gap between equilibrium and optimal product differentiation to widen. This observation thus allows us to predict that a policy that increases market transparency could also help reduce excessive differentiation and improve social welfare. For example, the government could enforce a policy requiring competing firms to provide more easily accessible information on the characteristics of their products, which would allow consumers to adapt to different products at lower costs and force firms to reduce unnecessary differentiation.

6 Concluding Remarks

This paper has proposed a model of R&D competition with process innovation and endogenous spillover. In particular, firms' choices of research tracks are related to their product configuration strategies. It is assumed that a firm's ability to appropriate its rival's R&D activity depends on how the firm positions its product. That is, the more differentiated a firm's products, the less knowledge spillover can be realized, and vice versa.

Using a Hotelling duopolistic model, it has been shown that, under certain conditions, firms may have the incentive to position their products closer to each other. However, in contrast to the existing literature on endogenous spillover, it has been shown here that firms do not necessarily maximize or minimize the information flow that results from knowledge spillover. Firms' equilibrium location pattern reflects the trade-off they face between the benefit from softened price competition (by furthering product differentiation) and the benefit from softened R&D competition (by reducing differentiation).

It has been shown that the equilibrium level of product differentiation depends on the extent of consumer heterogeneity, the cost of R&D and the sensitivity of the spillover rate to the level of differentiation, among other factors. A number of testable hypotheses may be derived from our analysis. Empirical studies that assess firms' activities under differing market conditions could further ascertain the nature of firms' incentives to conduct R&D related activities and the impact of various environmental factors on their ability to gain from the knowledge spillover. The above results may provide more useful insights on choosing and implementing appropriate policy instruments if the predictions can be tested based on empirical observations.

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